■ Chapter 2 Review Exercises

(pp. 91–93)

1. $\lim_{x\to-2}(x^3-2x^2+1)=(-2)^3-2(-2)^2+1=-15$

2. $\lim_{x\to-2}\dfrac{x^2+1}{3x^2-2x+5}=\dfrac{(-2)^2+1}{3(-2)^2-2(-2)+5}=\dfrac{5}{21}$

3. No limit, because the expression $\sqrt{1-2x}$ is undefined for values of x near 4.

4. No limit, because the expression $\sqrt[4]{9-x^2}$ is undefined for values of x near 5.

5. $\lim_{x\to0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}=\lim_{x\to0}\dfrac{2-(2+x)}{2x(2+x)}=\lim_{x\to0}\dfrac{-x}{2x(2+x)}$

$=\lim_{x\to0}\left(-\dfrac{1}{2(2+x)}\right)=-\dfrac{1}{2(2+0)}=-\dfrac{1}{4}$

6. $\lim_{x\to\pm\infty}\dfrac{2x^2+3}{5x^2+7}=\lim_{x\to\pm\infty}\dfrac{2x^2}{5x^2}=\dfrac{2}{5}$

7. An end behavior model for $\dfrac{x^4+x^3}{12x^3+128}$ is $\dfrac{x^4}{12x^3}=\dfrac{1}{12}x$.

Therefore:

9. Multiply the numerator and denominator by $\sin x$.

$\lim_{x\to0}\dfrac{x\csc x+1}{x\csc x}=\lim_{x\to0}\dfrac{x+\sin x}{x}=\lim_{x\to0}\left(1+\dfrac{\sin x}{x}\right)$

$=\left(\lim_{x\to0}1\right)+\left(\lim_{x\to0}\dfrac{\sin x}{x}\right)=1+1=2$

10. $\lim_{x\to0}e^x\sin x=e^0\sin 0=1\cdot0=0$

11. Let $x=\dfrac{7}{2}+h$, where h is in $\left(0,\dfrac{1}{2}\right)$. Then

$\text{int}\,(2x-1)=\text{int}\left[2\left(\dfrac{7}{2}\right)+2h-1\right]=\text{int}\,(6+2h)=6,$

because $6+2h$ is in $(6, 7)$.

Therefore, $\lim_{x\to7/2^+}\text{int}\,(2x-1)=\lim_{x\to7/2^+}6=6.$

12. Let $x=\dfrac{7}{2}+h$, where h is in $\left(-\dfrac{1}{2},0\right)$. Then

$\text{int}\,(2x-1)=\text{int}\left[2\left(\dfrac{7}{2}\right)+2h-1\right]=\text{int}\,(6+2h)=5,$

because $6+2h$ is in $(5, 6)$.

Therefore, $\lim_{x\to7/2^-}\text{int}\,(2x-1)=\lim_{x\to7/2^-}5=5$

13. Since $\lim_{x\to\infty}(-e^{-x})=\lim_{x\to\infty}e^{-x}=0$, and $-e^{-x}\le e^{-x}\cos x\le e^{-x}$ for all x, the Sandwich Theore gives $\lim_{x\to\infty}e^{-x}\cos x=0$.

14. Since the expression x is an end behavior model for both $x+\sin x$ and $x+\cos x$, $\lim_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}=\lim_{x\to\infty}\dfrac{x}{x}=1.$

15. Limit exists.

16. Limit exists.

17. Limit exists.

18. Limit does not exist.

19. Limit exists.

20. Limit exists.

21. Yes

22. No

23. No

24. Yes

25. **(a)** $\lim_{x\to3^-}g(x)=1$

(b) $g(3)=1.5$

26. **(a)** $\lim_{x\to1^-}k(x)=1.5$

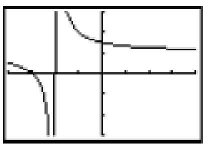
(b) $\lim_{x\to1^+}k(x)=0$

(c) $k(1)=0$

(d) No, since $\lim_{x\to1^-}k(x)\ne k(1)$

(e) k is discontinuous at $x=1$ (and at points not in the domain).

(f) No, the discontinuity at $x=1$ is not removable because the one-sided limits are different.

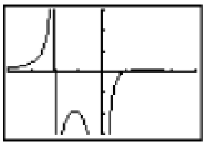
27. 

$[-4, 4]$ by $[-3, 3]$

(a) Vertical asymptote: $x=-2$

(b) Left-hand limit $=\lim_{x\to-2^-}\dfrac{x+3}{x+2}=-\infty$

Right-hand limit: $\lim_{x\to-2^+}\dfrac{x+3}{x+2}=\infty$

28. 

$[-4, 4]$ by $[-3, 3]$

(a) Vertical asymptotes: $x = 0, x = -2$

(b) At $x = 0$:

Left-hand limit $= \lim_{x \to 0^-} \frac{x-1}{x^2(x+2)} = -\infty$

Right-hand limit $= \lim_{x \to 0^+} \frac{x-1}{x^2(x+2)} = -\infty$

At $x = -2$:

Left-hand limit $= \lim_{x \to -2^-} \frac{x-1}{x^2(x+2)} = \infty$

Right-hand limit $= \lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty$

29. (a) At $x = -1$:

Left-hand limit $= \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (1) = 1$

Right-hand limit $= \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (-x) = 1$

At $x = 0$:

Left-hand limit $= \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$

Right-hand limit $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (-x) = 0$

At $x = 1$:

Left-hand limit $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-x) = -1$

Right-hand limit $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (1) = 1$

(b) At $x = -1$: Yes, the limit is 1.
At $x = 0$: Yes, the limit is 0.
At $x = 1$: No, the limit doesn't exist because the two one-sided limits are different.

(c) At $x = -1$: Continuous because $f(-1) =$ the limit.
At $x = 0$: Discontinuous because $f(0) \neq$ the limit.
At $x = 1$: Discontinuous because the limit does not exist.

30. (a) Left-hand limit $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} |x^3 - 4x|$

$= |(1)^3 - 4(1)| = |-3| = 3$

Right-hand limit $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2 - 2x - 2)$

$= (1)^2 - 2(1) - 2 = -3$

(b) No, because the two one-sided limits are different.

(c) Every place except for $x = 1$

(d) At $x = 1$

31. Since $f(x)$ is a quotient of polynomials, it is continuous and its points of discontinuity are the points where it is undefined, namely $x = -2$ and $x = 2$.

32. There are no points of discontinuity, since $g(x)$ is continuous and defined for all real numbers.

33. (a) End behavior model: $\frac{2x}{x^2}$, or $\frac{2}{x}$

(b) Horizontal asymptote: $y = 0$ (the x-axis)

34. (a) End behavior model: $\frac{2x^2}{x^2}$, or 2

(b) Horizontal asymptote: $y = 2$

35. (a) End behavior model: $\frac{x^3}{x}$, or x^2

(b) Since the end behavior model is quadratic, there are n horizontal asymptotes.

36. (a) End behavior model: $\frac{x^4}{x^3}$, or x

(b) Since the end behavior model represents a nonhorizontal line, there are no horizontal asymptote

37. (a) Since $\lim_{x \to \infty} \frac{x + e^x}{e^x} = \lim_{x \to \infty} \left(\frac{x}{e^x} + 1\right) = 1$, a right end behavior model is e^x.

(b) Since $\lim_{x \to -\infty} \frac{x + e^x}{x} = \lim_{x \to -\infty} \left(1 + \frac{e^x}{x}\right) = 1$, a left end behavior model is x.

38. (a, b) Note that $\lim_{x \to \pm\infty} \left(-\frac{1}{\ln|x|}\right) = \lim_{x \to \pm\infty} \left(\frac{1}{\ln|x|}\right) = 0$ and $-\frac{1}{\ln|x|} < \frac{\sin x}{\ln|x|} < \frac{1}{\ln|x|}$ for all $x \neq 0$.

Therefore, the Sandwich Theorem gives

$\lim_{x \to \pm\infty} \frac{\sin x}{\ln|x|} = 0$. Hence

$\lim_{x \to \pm\infty} \frac{\ln|x| + \sin x}{\ln|x|} = \lim_{x \to \pm\infty} \left(1 + \frac{\sin x}{\ln|x|}\right) = 1 + 0 = 1,$

so $\ln|x|$ is both a right end behavior model and a left end behavior model.

39. $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 + 2x - 15}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+5)}{x-3}$

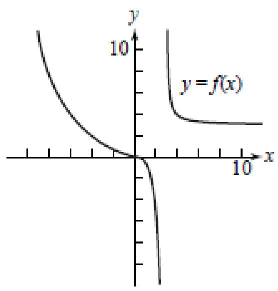
$= \lim_{x \to 3} (x + 5) = 3 + 5 = 8.$

Assign the value $k = 8$.

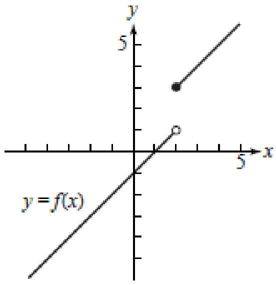
40. $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{x} = \frac{1}{2}(1) = \frac{1}{2}$

Assign the value $k = \frac{1}{2}$.

41. One possible answer:



42. One possible answer:



43. $\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{2-1}{\pi/2}=\frac{2}{\pi}$

44. $\lim_{h\to 0}\frac{V(a+h)-V(a)}{h}=\lim_{h\to 0}\frac{\frac{1}{3}\pi(a+h)^2H-\frac{1}{3}\pi a^2H}{h}$

$=\frac{1}{3}\pi H\lim_{h\to 0}\frac{a^2+2ah+h^2-a^2}{h}$

$=\frac{1}{3}\pi H\lim_{h\to 0}(2a+h)$

$=\frac{1}{3}\pi H(2a)$

$=\frac{2}{3}\pi aH$

45. $\lim_{h\to 0}\frac{S(a+h)-S(a)}{h}=\lim_{h\to 0}\frac{6(a+h)^2-6a^2}{h}$

$=\lim_{h\to 0}\frac{6a^2+12ah+6h^2-6a^2}{h}$

$=\lim_{h\to 0}(12a+6h)$

$=12a$

46. $\lim_{h\to 0}\frac{y(a+h)-y(a)}{h}$

$=\lim_{h\to 0}\frac{[(a+h)^2-(a+h)-2]-(a^2-a-2)}{h}$

$=\lim_{h\to 0}\frac{a^2+2ah+h^2-a-h-2-a^2+a+2}{h}$

$=\lim_{h\to 0}\frac{2ah+h^2-h}{h}$

$=\lim_{h\to 0}(2a+h-1)$

$=2a-1$

47. (a) $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{[(1+h)^2-3(1+h)]-(-2)}{h}$

$=\lim_{h\to 0}\frac{1+2h+h^2-3-3h+2}{h}$

$=\lim_{h\to 0}(-1+h)$

$=-1$

(b) The tangent at P has slope -1 and passes through $(1,-2)$.
$y=-1(x-1)-2$
$y=-x-1$

(c) The normal at P has slope 1 and passes through $(1,-2)$.
$y=1(x-1)-2$
$y=x-3$

48. At $x=a$, the slope of the curve is

$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{[(a+h)^2-3(a+h)]-(a^2-3a)}{h}$

$=\lim_{h\to 0}\frac{a^2+2ah+h^2-3a-3h-a^2+3a}{h}$

$=\lim_{h\to 0}\frac{2ah-3h+h^2}{h}$

$=\lim_{h\to 0}(2a-3+h)$

$=2a-3$

The tangent is horizontal when $2a-3=0$, at $a=\frac{3}{2}$ $\left(\text{or } x=\frac{3}{2}\right)$. Since $f\left(\frac{3}{2}\right)=-\frac{9}{4}$, the point where this occurs is $\left(\frac{3}{2},-\frac{9}{4}\right)$.

49. (a) $p(0)=\frac{200}{1+7e^{-0.1(0)}}=\frac{200}{8}=25$

Perhaps this is the number of bears placed in the reserve when it was established.

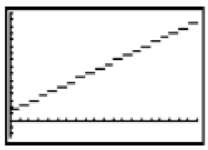
(b) $\lim_{t\to\infty}p(t)=\lim_{t\to\infty}\frac{200}{1+7e^{-0.1t}}=\frac{200}{1}=200$

(c) Perhaps this is the maximum number of bears which the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.

50. (a) $f(x)=\begin{cases}3.20-1.35\operatorname{int}(-x+1), & 0<x\le 20\\ 0, & x=0\end{cases}$

(Note that we cannot use the formula $f(x)=3.20+1.35\operatorname{int}x$, because it gives incorrect results when x is an integer.)

(b)



$[0, 20]$ by $[-5, 32]$

f is discontinuous at integer values of x: 0, 1, 2, ..., 19.

51. (a) Cubic: $y=-1.644x^3+42.981x^2-254.369x+300.232$

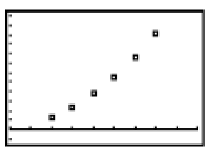
Quartic: $y=2.009x^4-102.081x^3+1884.997x^2-14918.180x+43004.464$

(b) Cubic: $-1.644x^3$, predicts spending will go to 0

Quartic: $2.009x^4$, predicts spending will go to ∞

52. Let $A=\lim_{x\to c}f(x)$ and $B=\lim_{x\to c}g(x)$. Then $A+B=2$ and $A-B=1$. Adding, we have $2A=3$, so $A=\frac{3}{2}$, whence $\frac{3}{2}+B=2$, which gives $B=\frac{1}{2}$. Therefore, $\lim_{x\to c}f(x)=\frac{3}{2}$ and $\lim_{x\to c}g(x)=\frac{1}{2}$.

53. (a)



$[3, 12]$ by $[-2, 24]$

(b)

Year of Q	Slope of PQ
1995	$\frac{20.1 - 2.7}{2000 - 1995} = 3.48$
1996	$\frac{20.1 - 4.8}{2000 - 1996} = 3.825$
1997	$\frac{20.1 - 7.8}{2000 - 1997} = 4.1$
1998	$\frac{20.1 - 11.2}{2000 - 1998} = 4.45$
1999	$\frac{20.1 - 15.2}{2000 - 1999} = 4.9$

(c) Approximately 5 billion dollars per year.

(d) $y = 0.3214x^2 - 1.3471x + 1.3857$

$$\begin{aligned}
\lim_{h\to 0} \frac{y(10+h) - y(10)}{h} &= \lim_{h\to 0} \frac{[0.3214(10+h)^2 - 1.3471(10+h) + 1.3857] - [0.3214(10)^2 - 1.3471(10) + 1.3857]}{h} \\
&= \lim_{h\to 0} \frac{0.3214(20h + h^2) - 1.3471h}{h} \\
&= 0.3214(20) - 1.3471 \\
&\approx 5.081
\end{aligned}$$

The predicted rate of change in 2000 is about 5.081 billion dollars per year.